Precautionary Saving in the Large: \( n \)th-Degree Deteriorations in Return Risk

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Abstract

Previous research on the measurement of the strength of the precautionary saving motive has concentrated on reactions to an exogenous risk on future income. Complementing the work by Liu (2014), I derive a statement analogous to Ross’ (1981) comparative risk aversion for precautionary saving under return-risk increases. The main theorem involves a comparison based on precautionary premia, whose definition deals explicitly with the immediate endogeneity of risk exposure under return risk. I also define preference-intensity measures and state conditions for a representation of the comparative strength of the precautionary-saving motive equivalent to the main theorem. All comparisons apply to a wide range of definitions of risk increases.

Keywords: Intertemporal choice, prudence, precautionary saving, higher-order risk, expected utility, Ross

JEL classification: D91, D81

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1 Introduction

A central result of the theoretical precautionary-saving literature is that a zero-mean risk added to future income will raise an agent’s saving if and only if the third derivative of expected utility (EU) is positive (e.g., Drèze and Modigliani 1966, 1972, Leland 1968, Sandmo 1970, Rothschild and Stiglitz 1971). For this case of income risk, Kimball (1990) defines, in analogy to the Arrow-Pratt measures of risk aversion, preference-intensity and monetary measures to quantify the precautionary saving motive. An example of the former is the coefficient of absolute prudence (formed by the negative of the third over the second utility derivative), and an example of the latter is the equivalent precautionary premium (the sure reduction in wealth that has the same effect on optimal choice as adding a zero-mean risk).

Eeckhoudt and Schlesinger (2008) extend the preference conditions for risk-induced increases in saving to higher-order risk increases. In addition to income risk, the authors treat the case with a risky saving return. Motivated by that paper, Liu (2014) adapts Kimball’s analysis of equivalent precautionary premia to higher-order increases in income risk. His preference conditions to compare the strength of precautionary reactions rely on the Liu and Meyer (2013b) concept of \((n/m)\)th-degree Ross more risk aversion and refer to the \((n+1)\)th and second EU derivatives. However, Liu focuses entirely on income risk.

The problem with return risk is of independent interest.\(^1\) It is more intricate, however, because risk exposure depends directly on the level of the endogenous choice variable, viz. saving. To treat saving under return risk, I start from the risk-premium concept that Briys et al. (1989) define for the case where the risk on wealth depends on a choice variable. According to Briys et al., the premium arises by comparing the utility evaluated under the optimal choice without risk and the EU evaluated at the optimal choice under risk. Adapting this concept to the intertemporal case allows me to define precautionary premia that reflect the strength of the precautionary saving reaction to increases in return risk.

\(^1\)In its structure, the problem with return risk resembles production choices under output or price risk or the labor/leisure decision under wage-rate risk. I do not explicitly treat the latter cases here.
also of higher order. In analogy to Ross (1981), I formulate a theorem that states the conditions under which the strength of the precautionary reactions to return-risk increases under two utility functions can be compared in three different ways, including precautionary premia. Starting from concepts in Liu and Meyer (2013a,b), I define, in addition, preference-intensity measures which allow to compare the strength of the precautionary reactions under only slightly stronger conditions on preferences. All results hold for various definitions of variations in risk, including Ekern risk increases.

Section 2 presents the two-period consumption/saving model I start from and introduces basic concepts for the analysis. Section 3 provides the main theorem and analysis. Section 4 defines generalized Liu and Meyer measures and applies them to compare the strength of precautionary reactions. Section 5 concludes. Proofs, which do not appear in the text, are gathered in the Appendix.

2 Model and Basic Concepts

Consider the following two-period consumption/saving model. Individual $u$ chooses saving $s_1$ out of first-period income $y_1$ such as to maximize the intertemporal utility objective

$$u (y_1 - s_1) + \beta E_1 u (y_2 + s_1 \tilde{R}_2)$$

where $u$ is the agent’s increasing, concave, and prudent per-period felicity function and $\beta$ is the utility discount factor. Period-two consumption is composed of income $y_2$ and saving with interest, where $R_2$ represents the gross return on any amount saved. Risk, indicated by a tilde, enters through $\tilde{R}_2$. According to the first-order condition

$$u' (c_1) = \beta E_1 \left[ u' (\tilde{c}_2) \tilde{R}_2 \right]$$

where $c_1$ is the agent’s consumption in period 1.
the agent will choose saving such that the marginal utility from foregoing consumption in
period 1 (i.e., saving a marginal amount) is to equal to the expected discounted marginal
utility from consuming instead in period 2.

The return-risk increases considered in the following refer to the \( n^{th} \)-degree the \( \ell \) first
moments preserving stochastic dominance (\( n-\ell \)-MPSD) order of Liu (2014).

**Definition 1 \((n^{th}\)-degree \( \ell\)-first-Moments-Preserving Stochastic Dominance)\)** For
any given integer \( \ell \) with \( 1 \leq \ell \leq n - 1 \), \( \tilde{x}_l \) dominates \( \tilde{x}_h \) in the \( n-\ell \)-MPSD if and only if
\( \tilde{x}_l \preceq_{nSD} \tilde{x}_h \) and \( E(\tilde{x}_l^j) = E(\tilde{x}_h^j) \) for \( j = 1, \ldots, \ell \).

\( n \) represents in Definition 1 the stochastic-dominance order, while \( \ell \) describes how many of the
first moments are invariant between the two random variables. The \( n-\ell \)-MPSD order is, thus,
a special case of a \( n \)-concave order (e.g., Denuit and Eeckhoudt 2010). Definition 1 covers
as special cases, in turn, \( n^{th} \)-degree mean-preserving stochastic dominance of Denuit and
Eeckhoudt (2013) for \( \ell = 1 \) and Ekern (1980) risk increases for \( \ell = n - 1 \). Thus, 2-1-MPSD
describes, for example, mean-preserving spreads (Rothschild and Stiglitz 1970), 3-2-MPSD
describes increases in downside risk (Menezes et al. 1980), and 4-3-MPSD describes increases
in outer risk (Menezes and Wang 2005).

The analysis below will also use the Liu and Meyer (2013b) concept of \((n/m)^{th}\)-degree
Ross more risk aversion. For its definition, I refer to the class \( U_{k-cv}^{S} \) of all \( k \)-concave functions
\( f \) on \( S \), defined as the set \( \{ f \mid (-1)^k f^{(k)}(z) \leq 0 \text{ for some } k \in \mathbb{N} \text{ and } z \in S \subseteq \mathbb{R} \} \), where \( f^{(k)}(.) \)
stands for the \( k^{th} \) derivative of \( f \).

**Definition 2 \(((n/m)^{th}\)-Degree Ross More Risk Aversion)\)** For two utility functions
\( u(x), v(x) \in U_{n-cv}^{D_x} \times U_{m-cv}^{D_x} \), \( n > m \), \( u(x) \) is \((n/m)^{th}\)-degree Ross more risk-averse than
\( v(x) \) if and only if there exists a \( \lambda > 0 \) such that
\[
\frac{u^{(n)}(x_a)}{v^{(n)}(x_a)} \geq \lambda \geq \frac{u^{(m)}(x_b)}{v^{(m)}(x_b)} \text{ for all } x_a, x_b \in D_x \subseteq \mathbb{R}^+_0.
\]

As is well known, as applied to intertemporal choice EU curvature represents jointly risk
and intertemporal preference components. Even if the intertemporal risk reaction seems to especially depend on intertemporal preferences (e.g., Bostian and Heinzl 2016), I stick in this paper to the conventional risk aversion terminology when referring to EU curvature. Definition 2 has a number of applications in the literature. For static choice, it covers, for example, the higher-order Ross coefficients involving the $n^{th}$ over the first EU derivative (e.g., Jindapon and Neilson 2007, Demui and Eeckhoudt 2010). In Liu (2014), the conditions describing the strength of the precautionary saving motive as associated with an increase in $n^{th}$-order income risk use the $(n + 1)^{th}$ and the second EU derivatives.

When the size of risk exposure is immediately endogenous to the optimal choice, as under return risk, measuring the intensity of risk responses is more complicated than when the risk is fully exogenous, as in the classic case with income risk. For a risk that is endogenous, in the sense that risk exposure depends directly on the choice of the endogenous variable $\alpha$, Briys et al. (1989) define the risk premium $\bar{P}$ of individual $f$ by:

$$f(w_0 + w(\bar{\alpha}, E\bar{x}) - \bar{P}) = Ef(w_0 + w(\alpha^*, \bar{x}))$$

where $w_0$ is the agent’s initial wealth and $w(\alpha, \bar{x})$ is the agent’s endogenous wealth component, with $\bar{\alpha}$ being the optimal choice when the risk is replaced by its expectation and $\alpha^*$ being the choice maximizing EU under risk $\bar{x}$. Defined in this way, $\bar{P}$ preserves the desirable properties of the corresponding Arrow-Pratt concept for fully exogenous risk: it is positive for risk averters and increases when risk increases in the sense of a mean-preserving spread.\(^2\)

As adapted to the consumption/saving model (1), the risk premium $\pi_f^{r_2}$ of individual $f$ associated with the change of the net return from $\bar{r}_{2,l}$ to $\bar{r}_{2,h}$ derives, thus, by comparing

$$E_1f(y_2 + s_1^{l}\bar{R}_{2,l} - \pi_f^{r_2}) = E_1f(y_2 + s_1^{h}\bar{R}_{2,h})$$  \(3\)

\(^2\)Both properties need not be preserved if in lieu of $\bar{\alpha}$ the left-hand side of the defining equation is evaluated at $\alpha^*$, too (as, e.g., in Drèze and Modigliani 1972).
where \(s_1^i, i = l, h\), is the optimal saving for return \(\tilde{r}_{2,i}\) from first-order condition (2),

\[
f'(y_1 - s_1^i) = \beta E_1 \left[ f' \left( y_2 + s_1^i \tilde{R}_{2,i} \right) \tilde{R}_{2,i} \right]
\]  

(4)

Accordingly, \(\pi^{r_2}\) corresponds to the safe reduction in period-two consumption \(\tilde{c}_2^*\) evaluated at optimal saving \(s_1^i\) for the reference return \(\tilde{r}_{2,l}\) that has the same effect on future EU as the change from \(\tilde{r}_{2,l}\) to \(\tilde{r}_{2,h}\).

The equivalent precautionary premium \(\theta^{r_2}\) has an analogous interpretation involving just, instead of future EU, future expected marginal utility. For individual \(f\), \(\theta^{r_2}\) arises by comparing first-order condition (4) evaluated for \(\tilde{r}_{2,h}\) with that condition but expected future marginal utility evaluated for \(\tilde{r}_{2,l}\) and period-two consumption \(\tilde{c}_2^l\) subtracted by the safe amount \(\theta^{r_2}\), such as to compensate for the variation in expected future marginal utility due to exposure to the higher return risk:

\[
f'(y_1 - s_1^{*h}) = \beta E_1 \left[ f' \left( y_2 + s_1^{*l} \tilde{R}_{2,l} - \theta^{r_2}_f \tilde{R}_{2,l} \right) \tilde{R}_{2,l} \right]
\]

As a consequence, the defining equation for \(\theta^{r_2}\) is

\[
E_1 \left[ f' \left( y_2 + s_1^{*l} \tilde{R}_{2,l} - \theta^{r_2}_f \tilde{R}_{2,l} \right) \tilde{R}_{2,l} \right] = E_1 \left[ f' \left( y_2 + s_1^{*h} \tilde{R}_{2,h} \right) \tilde{R}_{2,h} \right]
\]  

(5)

Note that the premia \(\pi^{r_2}_f\) and \(\theta^{r_2}_f\) as defined in equations (3) and (5) deal with increases in (return) risk, and not with the complete elimination of risk like the Briys et al. risk premium under immediately endogenous risk exposure or, similarly, the Arrow-Pratt risk-aversion and Kimball (1990) precautionary premia for fully exogenous risks.

### 3 Main Theorem

Eeckhoudt and Schlesinger (Proposition 2) show that an EU agent saves more in response
to an \( n^{th} \)-order stochastic dominance (NSD) deterioration in return risk if and only if the coefficient of partial \((k + 1)^{th}\)-order risk aversion exceeds \( k \) for all \( k = 1, 2, \ldots, n \). The following lemma, which their proposition implies, adapts this result to deteriorations of return risk in the \( n-\ell \)-MPSD order.

**Lemma 1** Let \( s^*_{R_{2,l}} \) and \( s^*_{R_{2,h}} \) be the optimal saving choices from condition (2) under the return lotteries \( \tilde{R}_{2,l} \) and \( \tilde{R}_{2,h} \), respectively. The following statements are equivalent:

1. \( s^*_{R_{2,h}} \geq s^*_{R_{2,l}} \), if \( -s_1 R_2 u^{(k+1)}(y_2+s_1 R_2) / u^{(k)}(y_2+s_1 R_2) \geq k \) for all \( k = \ell, \ell + 1, \ldots, n \).

2. \( \tilde{R}_{2,l} \) dominates \( \tilde{R}_{2,h} \) via \( n-\ell \)-MPSD.

The saving reaction to return risk is characterized by the interplay of a positive precautionary effect and a negative substitution effect. Whether or not saving increases in response to a return-risk increase depends on whether the precautionary or the substitution effect dominates. Technically, the level condition on the partial risk-aversion coefficient in Lemma 1 decides which effect prevails. Because the conditions in the lemma are necessary and sufficient, it predicts similarly a decrease in the agent’s saving in response to a risk increase if the level condition is not fulfilled. According to the simulation results of Bostian and Heinzel (2016), this can be a common case.

Because \( \theta^{r_2} \) refers to the total saving response, the precautionary premium under return risk can be positive or negative, reflecting a dominating precautionary or substitution effect, respectively. The following theorem has, accordingly, two formulations. Stated in analogy to Ross (1981: Theorem 3), it extends Theorem 3 in Liu (2014) to return risk.

**Theorem 1** Suppose \( n \geq 2 \) and \( 1 \leq \ell \leq n - 1 \). For two utility functions \( u, v \) in the class \( U_{k-v}^{D_2} \) of all \( k \)-concave functions for \( k = 1, 2 \) and \( \ell + 1, \ldots, n + 1 \), for which saving increases (decreases) in response to a return-risk increase and involving the same saving level \( s_1^{th} \) under the reference return \( \tilde{r}_{2,h} \), the following conditions are equivalent:

(i) \((-u')\) is \( k^{th} \)-degree Ross more risk averse than \((-v')\) for all \( k = \ell + 1, \ldots, n \).
(ii) There exist \( \lambda > 0 \) and \( \phi'(x) \) with \( \phi''(x) \geq 0 \) and \( \phi'(x) \in U^{D_k}_{(k+1)-cv} \) for \( k = \ell + 1, \ldots, n \) and all \( x \in D_x \subset \mathbb{R}^+_0 \) such that \( u'(x) = \lambda v'(x) + \phi'(x) \).

(iii) \( |\theta_{u}^{\ell} \geq \theta_{v}^{\ell}| \) for all \( \tilde{r}_{2,l}, \tilde{r}_{2,h} \) with \( \tilde{r}_{2,l} \geq_{n-\ell-MPSD} \tilde{r}_{2,h} \) and \( \theta_{f}^{\ell} \) as defined in equation (5) for \( f \in \{u, v\} \), if the following condition holds:

\[
\frac{u'(y_1 - s_{1,u}^{th}) - u'(y_1 - s_{1,u}^{th})}{v'(y_1 - s_{1,v}^{th}) - v'(y_1 - s_{1,v}^{th})} \geq \lambda. \tag{6}
\]

The theorem provides two ways to compare the strength of the precautionary saving motive when facing a return-risk increase in the n-\( \ell \)-MPSD order. Following Definition 2, Statement (i) amounts to the same as requiring \( u \) to be \(((k+1)/2)^{th}\)-degree Ross more risk-averse than \( v \) for \( k = \ell + 1, \ldots, n \) and involves, thus, conditions which compare curvature properties of the two utility functions. Moreover, Statement (iii) posits that, equivalently, agent \( u \) with a positive (negative) saving response to a return-risk increase, has a uniformly larger (smaller) precautionary premium than agent \( v \), if the risk reaction of \( u \) is sufficiently stronger than the risk reaction of \( v \) (so that condition (6) is fulfilled).

An important special case of deteriorations in the n-\( \ell \)-MPSD order are Ekern risk increases, which arise when \( \ell = n - 1 \). Corollary 1 applies the above theorem to this case.

**Corollary 1** Suppose \( n \geq 2 \) and that \( u, v \) are two utility functions in the class \( U^{D_k}_{k-cv} \) of all \( k \)-concave functions for \( k = 1, 2 \) and \( n+1 \), for which saving increases (decreases) in response to a return-risk increase and involving the same saving level \( s_{1}^{th} \) under the reference return \( \tilde{r}_{2,h} \). Then, given that condition (6) holds, \( \theta_{u}^{\ell} \geq \theta_{v}^{\ell} \) for all \( n^{th}\)-degree risk increases in the return on saving if and only if \((-u')\) is \( n^{th}\)-degree Ross more risk averse than \((-v')\) or, equivalently, there exists a constant \( \lambda > 0 \) such that

\[
\frac{u^{(n+1)}(x_a)}{v^{(n+1)}(x_a)} \geq \lambda \geq \frac{u''(x_b)}{v''(x_b)} \quad \text{for all } x_a, x_b \in D_x \subset \mathbb{R}^+_0.
\]
4 Representation Based on Preference Coefficients

The above preference conditions to compare the precautionary saving responses to return-risk increases start from the curvature properties stipulated for the two utility functions in Definition 2 of \((n/m)\)th-degree Ross more risk aversion. Similar to Ross (1981), however, none of the conditions involves coefficients measuring immediately preference intensity. The following definition adapts concepts from Liu and Meyer (2013a,b) to fill this gap. The subsequent theorem provides then the conditions for an alternative statement of the above preference conditions using coefficients of precautionary intensity.

Liu and Meyer (2013a) introduce a concavity measure which evaluates in the Arrow-Pratt coefficient of risk aversion for some utility function \(f(x)\), with \(x \in [a, b] \subset \mathbb{R}\), marginal utility at the fixed value \(x_0 \in [a, b]\). The authors show that, notably due to the monotonicity of \(f'(.)\) in its argument, the arising normalized measure

\[
C_f(x; x_0) \equiv -\frac{f''(x)}{f'(x_0)}
\]

allows for an alternative characterization of Ross more risk aversion when choosing \(x_0 = a\). Liu and Meyer (2013b), moreover, define for an \(n\)-times differentiable function \(f \in \mathcal{U}_{m-cv}^S\) the local \((n/m)\)th-degree absolute risk aversion measure as:

\[
A_{(n/m)}^f(x) = \frac{(-1)^n-1 f^{(n)}(x)}{(-1)^m-1 f^{(m)}(x)}
\]

**Definition 3 (Generalized Liu and Meyer Measure)** For an \(n\)-times differentiable utility function \(f(x) \in \mathcal{U}_{m-cv}^{[a,b]}\), with \(n > m \geq 1\) and \([a, b] \subset D_x \subset \mathbb{R}_0^+\), the generalized Liu and Meyer measure of \((n/m)\)th-degree Ross more risk aversion is defined as

\[
C_{(n/m)}(x; a) = (-1)^{n-m} \frac{f^{(n)}(x)}{f^{(m)}(a)}
\]

(7)
The following theorem shows how the comparative conditions for the precautionary reactions to a return risk increase between the agents $u$ and $v$ in Theorem 1 can be complemented by a representation based on generalized Liu and Meyer measures.

**Theorem 2** Given that $u$ is more Kimball-prudent than $v$, i.e., $-\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)}$ for all $x \in [a, b] \subset D_x$, then the following condition is equivalent to Statements (i)–(iii) in Theorem 1:

$$C_{((k+1)/2),u}(x;a) \geq C_{((k+1)/2),v}(x;a) \text{ for all } k = \ell + 1, \ldots, n$$

with $C_{((k+1)/2),f}(x;a)$, for $f \in \{u, v\}$, as defined in equation (7).

For $\ell = n - 1$, Theorem 2 covers similarly the case of Ekern risk increases, as in Corollary 1. Note, moreover, that an analogous theorem holds for the case of income-risk increases as treated by Liu (2014). While Liu and Meyer (2013a) find an immediate equivalence between Ross more risk aversion and a comparative statement based on their concavity measure for the static reaction to a risk increase, the application of generalized Liu and Meyer measures in intertemporal choice requires, in addition, the that Ross more risk-averse agents is more prudent in the sense of Kimball (1990).

### 5 Conclusion

The measurement of the strength of the precautionary saving motive has concentrated on reactions to an exogenous risk on future income. Complementing the work of Liu (2014) on increases in income risk, I derive a statement analogous to Ross’ (1981) comparative risk aversion for precautionary saving under return-risk increases. The main theorem involves a comparison based on precautionary premia, which I define starting from the approach of Briys et al. (1989). In contrast to situations where optimal choices have no direct influence on risk exposure, such as the classic case with income risk, the immediate endogeneity of risk exposure under return risk requires to control for the (different) levels of optimal saving in the
reference case and the situation with increased risk. For a prudent agent, the saving reaction to a return-risk increase may be positive or negative depending on whether the precautionary or the substitution effect dominates. The precautionary premium, which refers to the total saving reaction, will accordingly be positive or negative.

Building on Liu and Meyer (2013a,b), I also define preference-intensity measures and state conditions for an equivalent representation of the comparative statements in the main theorem based on these coefficients. An analogous representation holds for income-risk increases. The strengths and weaknesses discussed, for example, by Pratt (1990) and Liu and Meyer (2013a) for such Ross-type measures, which are necessarily measures in the large, apply naturally also here. Together with Liu (2014), now a complete set of premia and preference-intensity measures are available, which conceptually allow to compare the precautionary motives underlying the saving conditions in Eeckhoudt and Schlesinger (2008).

In further work, it will be interesting to see these measures applied. Given the distinct roles of risk and intertemporal preferences and the importance of recursive preferences in intertemporal choice (e.g., Epstein and Zin 1989), it may be worthwhile to workout corresponding conditions for recursive utility and to compare the results with the EU case. Desirable extensions concern, moreover, the treatment of other cases where optimal choice directly (co-)determines risk exposure, such as production and consumption/leisure choices but also self-protection or self-insurance.
References


Appendix

A  Proof of Theorem 1

(i) ⇒ (ii). By Definition 2, there exist $\lambda_k$ for all $k = \ell + 1, \ldots, n$ such that

$$\frac{(-u')^{(k)}(x_a)}{(-v')^{(k)}(x_a)} \geq \lambda_k \geq \frac{(-u')^{(k)}(x_b)}{(-v')^{(k)}(x_b)}$$

for all $x_a, x_b \subset D_x$.

Define $\lambda \equiv \min \{\lambda_k\}_{k+1} > 0$. Then, for $k = \ell + 1, \ldots, n$, $u^{(k+1)}(x_a) \geq \lambda \geq u^{(k)}(x_b)$ for all $x_a, x_b \subset D_x$. Define $\phi'(x)$ by $u'(x) = \lambda \nu'(x) + \phi'(x)$. Thus, $\phi''(x) = u''(x) - \lambda \nu''(x) \geq 0$ and $(-1)^k \phi^{(k+1)}(x) = (-1)^k [u^{(k+1)}(x) - \lambda \nu^{(k+1)}(x)] \geq 0$ for all $x \in D_x$ and $k = \ell + 1, \ldots, n$.

(ii) ⇒ (iii). Consider first the case where saving under $u$ and $v$ increases in response to a given return-risk increase (i.e., the saving-increase condition in Lemma 1 is fulfilled for all $k = \ell, \ell + 1, \ldots, n$), so that $\theta_u^r, \theta_v^r \geq 0$ and $s^t_{1,u} \leq s^t_{1,v} \leq s^h_{1,u} \equiv s^h_{1,v}$. With $u'(x) = \lambda \nu'(x) + \phi'(x), \phi'(x) \in \mathcal{U}_{(k+1)-cv}^{D_x}$ for $k = \ell + 1, \ldots, n$, and $\phi''(x) \geq 0, \theta_u^r \geq \theta_v^r$ is implied by

$$E_1 \left[ u'(y_2 + s^t_{1, u} \tilde{R}_{2,l} - \theta_v^r) \tilde{R}_{2,l} \right] = E_1 \left[ u'(y_2 + s^h_{1, u} \tilde{R}_{2,h} \tilde{R}_{2,l} \right]$$

$$= \lambda E_1 \left[ \nu'(y_2 + s^h_{1, u} \tilde{R}_{2,h} \tilde{R}_{2,l} \right] + E_1 \left[ \phi'(y_2 + s^h_{1, u} \tilde{R}_{2,h} \tilde{R}_{2,l} \right]$$

$$\geq \lambda E_1 \left[ \nu'(y_2 + s^h_{1, u} \tilde{R}_{2,h}) \tilde{R}_{2,h} \right] + E_1 \left[ \phi'(y_2 + s^h_{1, u} \tilde{R}_{2,h}) \tilde{R}_{2,l} \right]$$

$$\geq \lambda E_1 \left[ \nu'(y_2 + s^h_{1, u} \tilde{R}_{2,h}) \tilde{R}_{2,h} \right] + E_1 \left[ \phi'(y_2 + s^h_{1, u} \tilde{R}_{2,h}) \tilde{R}_{2,l} \right]$$

if condition (6) holds. In system (8), the first inequality follows, because the NSD equivalence and Proposition 2 in Eeckhoudt and Schlesinger hold in an analogous way for some function.
so that increase (marginal utility according to the appropriate version of first-order condition (2)).

Lemma 1 above), the second inequality follows from \( \phi''(x) \geq 0 \) and \( \theta_r^2 \geq 0 \), and the third inequality is implied by \( v''(x) \leq 0 \) and \( s_{1,v}^l \geq s_{1,u}^l \). Condition (6) arises by comparing in system (8) the difference between the two sides of the first inequality with the supremum for \( \theta_r^2 \to 0 \) of the difference between the two sides of the third inequality, noting that, by definition, \( \phi'(x) = u'(x) - \lambda v'(x) \) and substituting finally in each case present for future marginal utility according to the appropriate version of first-order condition (2).

In the other case, where saving under \( u \) and \( v \) decreases in response to a given return-risk increase (i.e., the saving-increase condition in Lemma 1 is violated for all \( k = \ell, \ell + 1, \ldots, n \)), so that \( \theta_u^2, \theta_v^2 \leq 0 \) and \( s_{1,u}^h \equiv s_{1,v}^h \leq s_{1,v}^l \leq s_{1,u}^l \), \( \theta_r^2 \leq \theta_u^2 \) is implied by

\[
E_1 \left[u'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_u^2) \tilde{R}_{2,t}\right] = E_1 \left[u'(y_2 + s_{1,u}^h \tilde{R}_{2,h}) \tilde{R}_{2,h}\right]
\]

\[
= \lambda E_1 \left[u'(y_2 + s_{1,u}^h \tilde{R}_{2,h}) \tilde{R}_{2,h}\right] + E_1 \left[\phi'(y_2 + s_{1,u}^h \tilde{R}_{2,h}) \tilde{R}_{2,h}\right]
\]

\[
\leq \lambda E_1 \left[u'(y_2 + s_{1,u}^h \tilde{R}_{2,h}) \tilde{R}_{2,h}\right] + E_1 \left[\phi'(y_2 + s_{1,u}^l \tilde{R}_{2,l}) \tilde{R}_{2,l}\right]
\]

\[
= \lambda E_1 \left[u'(y_2 + s_{1,u}^h \tilde{R}_{2,h}) \tilde{R}_{2,h}\right] + E_1 \left[\phi'(y_2 + s_{1,u}^l \tilde{R}_{2,l}) \tilde{R}_{2,l}\right]
\]

\[
\leq \lambda E_1 \left[u'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_v^2) \tilde{R}_{2,t}\right] + E_1 \left[\phi'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_v^2) \tilde{R}_{2,t}\right]
\]

\[
\geq \lambda E_1 \left[u'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_v^2) \tilde{R}_{2,t}\right] + E_1 \left[\phi'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_v^2) \tilde{R}_{2,t}\right]
\]

\[
= E_1 \left[u'(y_2 + s_{1,u}^l \tilde{R}_{2,t} - \theta_v^2) \tilde{R}_{2,t}\right]
\]

if condition (6) holds. The inequalities as well as the sufficient condition (6) follow in an analogous way to before, only the direction of the inequalities is reversed because of the violation of the saving-increase condition in Lemma 1.

(iii) \(\Rightarrow\) (i). Consider first the case where saving under \( u \) and \( v \) increases in response to a given return-risk increase. From \( \theta_u^2 \geq \theta_v^2 \) for all \( \tilde{r}_{2,t}, \tilde{r}_{2,h} \) such that \( \tilde{r}_{2,t} \leq_{\ell-MPSD} \tilde{r}_{2,h} \) and \( \theta_f^2 \) defined as in (5) for \( f \in \{u, v\} \), it follows that \( \theta_u^2 \geq \theta_v^2 \) for all \( \tilde{r}_{2,t}, \tilde{r}_{2,h} \) such that \( \tilde{r}_{2,h} \).
is a $k^{th}$-degree Ekern risk increase over $\tilde{r}_{2,l}$ for all $k = \ell + 1, \ldots, n$. Based on an argument analogous to the one in Liu (2014) and on Definition 2, this implies that $(-u')$ has a stronger $k^{th}$-degree Ross intertemporal attitude than $(-v')$ for all $k = \ell + 1, \ldots, n$. In the second case, where saving under $u$ and $v$ decreases in response to a given return-risk increase, $\theta_{u}^{r2} \leq \theta_{v}^{r2}$ follows analogously. ■

B  Proof of Theorem 2

It is sufficient to prove the equivalence between the ordering of the generalized Liu and Meyer measures, denoted (iv), and Statement (i) in Theorem 1, given that, for all $x \in [a,b] \subset D_x$,

$$-\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)}. \quad (9a)$$

(i) $\Rightarrow$ (iv). Given (i), there exists, for all $x_a, x_b \in [a,b]$ and $k = \ell + 1, \ldots, n$, a $\lambda > 0$ such that,

$$\frac{u^{k+1}(x_a)}{v^{k+1}(x_a)} \geq \lambda \geq \frac{u''(x_b)}{v''(x_b)}. \quad (9b)$$

Let $x_b = a$. Then, for all $x \in [a,b]$ and $k = \ell + 1, \ldots, n$,

$$\frac{u^{k+1}(x)}{v^{k+1}(x)} \geq \frac{u''(a)}{v''(a)} \iff (-1)^{k-1} \frac{u^{k+1}(x)}{u''(a)} \geq (-1)^{k-1} \frac{v^{k+1}(x)}{v''(a)}. \quad (9b)$$

(iv) $\Rightarrow$ (i). Assume $C_{((k+1)/2)}(x; a) \geq C_{((k+1)/2)}(x; a)$ for all $x \in [a,b]$ and $k = \ell + 1, \ldots, n$, so that, for $x = x_a$, equivalently,

$$(-1)^{k-1} \frac{u^{k+1}(x_a)}{u''(a)} \geq (-1)^{k-1} \frac{v^{k+1}(x_a)}{v''(a)} \iff \frac{u^{k+1}(x_a)}{v^{k+1}(x_a)} \geq \frac{u''(a)}{v''(a)}.$$ 

By setting $\lambda \equiv \frac{u''(a)}{v''(a)}$, the first inequality in (9b) arises. The second inequality in (9b) holds because $\frac{u''(a)}{v''(a)}$ decreases in its argument under condition (9a). ■
The FOODSECURE project in a nutshell

Title
FOODSECURE – Exploring the future of global food and nutrition security

Funding scheme
7th framework program, theme Socioeconomic sciences and the humanities

Type of project
Large-scale collaborative research project

Project Coordinator
Hans van Meijl (LEI Wageningen UR)

Scientific Coordinator
Joachim von Braun (ZEF, Center for Development Research, University of Bonn)

Duration
2012 - 2017 (60 months)

Short description
In the future, excessively high food prices may frequently reoccur, with severe impact on the poor and vulnerable. Given the long lead time of the social and technological solutions for a more stable food system, a long-term policy framework on global food and nutrition security is urgently needed.

The general objective of the FOODSECURE project is to design effective and sustainable strategies for assessing and addressing the challenges of food and nutrition security.

FOODSECURE provides a set of analytical instruments to experiment, analyse, and coordinate the effects of short and long term policies related to achieving food security.

FOODSECURE impact lies in the knowledge base to support EU policy makers and other stakeholders in the design of consistent, coherent, long-term policy strategies for improving food and nutrition security.

EU Contribution
€ 8 million

Research team
19 partners from 13 countries

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